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**Patentanmeldung Nr.    Patent application No.    Demande de brevet n°**

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Der Präsident des Europäischen Patentamts;  
Im Auftrag

For the President of the European Patent Office

Le Président de l'Office européen des brevets  
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**R C van Dijk**

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**Blatt 2 der Bescheinigung**  
**Sheet 2 of the certificate**  
**Page 2 de l'attestation**

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## Method for determining the yaw angle of a satellite

The invention relates to a method for determining the yaw angle of a satellite.

In order to effectively control satellites, especially geostationary satellites, the exact orientation of the satellite has to be known. Apart from other values, the roll angle, the pitch angle and the yaw angle, which will be described in greater detail further below, have to be measured or estimated. Most geostationary, so called "three-axis stabilized" satellites are provided with sensors that allow to measure and actively control the roll and pitch angle. In this case, no sensor is provided for measuring the yaw angle. The yaw angle is usually estimated and controlled by means of the roll/yaw coupling that occurs throughout the orbit when an angular momentum bias is present on the spacecraft, for instance when a momentum wheel is spinning inside. Since this coupling is actually very light, a proper yaw angle estimation and correction takes several hours. Although these spacecrafts also are usually equipped with rate measurement assemblies like gyroscopes with a fast measurement response around all three axes, the latter are only used for autonomous attitude measurements over short periods of time for instance when the attitude is expected to be disturbed like during station keeping manoeuvres. The reason is that the integrated angle tends to drift away due to the presence of an inherent bias in the measured rates. In addition, when the rate measurement devices are gyroscopes the risk of mechanical wear leads the operators to turn them off whenever their use is not absolutely

required.

Even though these types of satellites usually control their yaw angle very acceptably without measuring it, there are cases when a fast yaw measurement is highly desirable like after an unexpected attitude disturbance or when not enough angular momentum bias is present to allow sufficient coupling between the roll and yaw angles, for instance when the momentum wheel has failed. A daily monitoring of the yaw angle profile is also useful to evaluate the health of the attitude control system.

Against this background the technical problem to be solved is to provide a method for determining the yaw angle of a satellite on the basis of sensor measurement signals readily available at the satellite, i.e. without the requirement of additional sensors.

This problem is solved by a method for determining the yaw angle of a satellite for which a cartesian coordinate system is definable having three axes a first of which is directed substantially tangential to the orbit of the satellite and defines a roll angle, a second of which is substantially perpendicular to the orbit plane of the satellite and defines a pitch angle, and a third of which is substantial radial to the orbit of the satellite and defines a yaw angle, the method comprising the steps of:

- measuring a first roll angle and/or a first pitch angle by means of a first sensor related to a first reference point;
  - measuring a second roll angle and/or a second pitch angle by means of a second sensor related to a second reference point which is different from the first reference point;
  - evaluating the first and/or second roll angles and the first and/or second pitch angles to determine the yaw angle of the satellite,
- as described in claim 1.

Advantageous and preferred embodiments are described in the subclaims and in the detailed description of preferred embodiments further below.

The method according to the invention allows to measure the yaw angle from the reading of two different sensors measuring the roll and/or pitch angles, provided that the reference point of the two sensors are not identical. The description is given basically for geostationary satellites but the invention can be applied directly to satellites which are stationary with respect to any star. The method can also be employed for non circular orbits. The method assumes that the orbit of the spacecraft is known at any time.

Generally speaking, the invention teaches a method for determining an unknown orientation angle of a satellite for which a coordinate system is definable having three axes which are arranged in an orthogonal trihedron, the method comprising the steps of measuring a first orientation angle defined by a first axis and/or a second orientation angle defined by a second axis by means of a first sensor related to a first reference point; measuring a third orientation angle defined by said first axis and/or a fourth orientation angle defined by said second axis by means of a second sensor related to a second reference point which is different from the first reference point; evaluating the first and/or second orientation angles and the third and/or fourth orientation angles to determine the unknown orientation angle of the satellite.

In the following, the method according to the invention and its principles will be explained in greater detail with reference to the drawing of which

Fig. 1 shows a view of an orbiting satellite in an earth orbit for illustrating a reference coordinate system.

Fig. 2 shows a view of the earth from orbiting satellite including geometric indicators for explaining roll and pitch angles with and without the presence of a yaw angle.

Fig. 3 shows a geometric relation for explaining the principles of the method according to the invention.

Fig. 4 shows a view of the earth from a orbiting satellite including geometric indicators for explaining the principles of the method according to the invention.

In order to define the position and orientation of a satellite, there is a need for the definition of a reference frame, i.e. a triplet of axes virtually attached to the body of the satellite, the latter being considered as infinitely rigid.

The following discussion will be limited to geostationary satellites. In this case, the location of the center of the frame (orbit) is less involved in the following development. It should be noted that in any case the orbit only affects the location of the reference points of the sensors on earth. So if the orbit is not geostationary, the location of the reference points on the earth deterministically vary throughout this orbit.

The three axes are named roll, pitch and yaw and are oriented as shown in Fig. 1, i.e.

roll:	along the velocity vector of the satellite (tangential to the orbit)
pitch:	perpendicular to the orbit plane of the satellite, directed towards South;
yaw:	nominally towards the center of the earth so as to close the tri-orthogonal right-hand oriented frame.



The orientation of the satellite, further called "attitude" can be specified in terms of rotations about these three axes so that one may speak of roll, pitch and yaw angles. Roll, Pitch and Yaw angles are actually «Euler angles». This means that they represent the orientation of a body with respect to a reference orientation by consecutive rotations around the corresponding axes. The final orientation depends on the order of rotations. In other words, if the yaw/pitch/roll order of rotations is picked, then starting from the reference frame of Fig.1, the final attitude is reached by first rotating the frame around the yaw axis, then rotating the frame around the new pitch axis, then around the new roll axis. The final orientation would not be the same if the rotations were performed, with the same magnitude, but in a different order. However, if the magnitude of the rotations are small enough, the final attitude becomes almost independent of the order of rotations. This approximation is often applied when the attitude errors of a properly controlled geostationary satellite are considered, since its pointing errors are very small.

Fig. 2 shows the earth seen from a satellite, as well as the directions P and R in which pitch and roll errors will move a beam sent by the satellite on the earth. The solid lines show the directions along which roll and pitch errors would be measured if there were no yaw error, the dotted lines give the same information but when some positive yaw error is present.

As an example and as shown in Fig. 2, points G and N locate distinct reference points of two sensors provided on-board the satellite. The reference point of each sensor is defined as the location it points to when the errors it reads are zero.

As an example, two sensors used on-board the ASTRA satellites are considered in the following, but different

kinds of sensors may be used as well. One sensor is an optical infrared earth sensor assembly (ESA) with the sub-nadir point N (center of the earth) as its reference point. The other sensor is a beacon sensor with the ground station G as its reference point. Each sensor issues roll and pitch angle attitude errors defining the difference between the direction it points to, its "boresight", and its reference point (identified by points G and N). The satellite transmits the telemetry values of the measured roll and pitch angles of both sensors to the ground station which records them for further processing and/or analysis. The roll and pitch errors of at least one of the sensors are also sent to the on-board processor for roll and pitch control.

It should be noted again that the method explained below extends to any kind of pair of sensors measuring roll and pitch angles or two linear combination of these angles, as long as the reference points G and N of the two sensors are different. In addition, the method can also readily be extended to a point N not being on the center of the earth.

The roll and pitch angles are often represented like planar coordinates as in Fig. 2. However, since these angles are actually Euler angles, this representation is only valid for small angles.

According to the definitions above, roll and pitch errors with a zero yaw error would show up as shown in Fig. 3, where R1, P1 and R2, P2 are the errors measured by the two sensors in roll and pitch. Of course, the length of the segment joining the boresights of the two sensors when there is an off-pointing (an attitude error) is the same as when there is no error (line joining points N and G).

If a yaw error  $y$  exists, as shown in Fig. 4, the direction in which roll and pitch errors are measured would be canted by  $y$ . In this situation, the readings of R1, P1 and R2, P2 are not equal anymore between the two sensors. This means

information about the yaw angle is captured in the difference between the readings of the two sensors.

In order to retain only the yaw information, and since  $R_1$  and  $P_1$  are assumed to be small angles, which means that a planar representation can be applied, the actual roll and pitch errors can be eliminated by geometrically translating  $R_2$  and  $P_2$  respectively by  $R_1$  and  $P_1$  ending up on point  $G'$  as shown in Fig. 5. When using the difference between the reading of the roll error on one hand and between the readings of the pitch error on the other hand two angle  $\Delta_p$  and  $\Delta_r$  can be defined:

$$(1) \quad \Delta_p = P_2 - P_1$$

$$(2) \quad \Delta_r = R_2 - R_1$$

In other words, the problem is now reduced to a pure yaw angle, resulting in the second sensor to read  $\Delta_p$  and  $\Delta_r$  namely the coordinates of point  $G'$ , and the first sensor to read zero.

Although not really necessary, it can be assumed that  $\Delta_p$  and  $\Delta_r$  are small angles. This slightly simplifies the considerations below. This assumption has been verified for all ASTRA spacecrafts since a yaw angle of the order of a degree leads to  $\Delta_p$  and  $\Delta_r$  which are about one tenth thereof.

In order to find the yaw angle provoking a  $\Delta_p$  and  $\Delta_r$ , that yaw angle has to be determined that would rotate point  $G$  to point  $G'$ , i.e. that would make the second sensor to read the errors  $\Delta_p$  and  $\Delta_r$ . Knowing that point  $G$  is defined by the consecutive pitch (azimuth) and roll (elevation) rotations that bring the center of the earth to the reference point of the second sensor and point  $G'$  is defined as the pitch and roll rotations that bring the center of the earth, as seen from the spacecraft, to a location where the second sensor reads  $\Delta_p$  and  $\Delta_r$ .

The following equations represent the rotations of a vector pointing from the spacecraft to the center of the earth: on one side, a pitch rotation (pitch\_GP) followed by a roll rotation (roll\_GP) with no yaw rotation which bring the center of the earth to G' and, on the other side, a pitch rotation (pitch\_G) followed by a roll rotation (roll\_G) which bring the center of the earth to G, and finally followed by a yaw rotation that brings point G to point G'.

$$(3) \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\text{roll\_GP}) & -\sin(\text{roll\_GP}) \\ 0 & \sin(\text{roll\_GP}) & \cos(\text{roll\_GP}) \end{bmatrix} \cdot \begin{bmatrix} \cos(\text{pitch\_GP}) & 0 & \sin(\text{pitch\_GP}) \\ 0 & 1 & 0 \\ -\sin(\text{pitch\_GP}) & 0 & \cos(\text{pitch\_GP}) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\text{yaw}) & -\sin(\text{yaw}) & 0 \\ \sin(\text{yaw}) & \cos(\text{yaw}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\text{roll\_G}) & -\sin(\text{roll\_G}) \\ 0 & \sin(\text{roll\_G}) & \cos(\text{roll\_G}) \end{bmatrix} \begin{bmatrix} \cos(\text{pitch\_G}) & 0 & \sin(\text{pitch\_G}) \\ 0 & 1 & 0 \\ -\sin(\text{pitch\_G}) & 0 & \cos(\text{pitch\_G}) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The above expression defines a system of three equations with only "yaw" as an unknown. For the ASTRA spacecrafts, the variables pitch\_G and roll\_G are actually the azimuth and elevation of the ground station (Betzdorf, Luxembourg) as seen from the spacecraft, while since Delta\_p and Delta\_r are small the angles pitch\_GP and roll\_GP can be defined as follows:

$$(4) \quad \text{pitch\_GP} = \text{pitch\_G} + \text{Delta\_p}$$

$$(5) \quad \text{roll\_GP} = \text{roll\_G} + \text{Delta\_r}$$

Developing system (3), it yields:

$$(6) \quad \sin(\text{pitch\_GP}) = \cos(\text{yaw}) * \sin(\text{pitch\_G}) + \sin(\text{yaw}) * \sin(\text{roll\_G}) * \cos(\text{pitch\_G})$$

$$(7) \quad -\sin(\text{roll\_GP}) \cos(\text{pitch\_GP}) = \sin(\text{yaw}) * \sin(\text{pitch\_G}) - \cos(\text{yaw}) * \sin(\text{roll\_G}) * \cos(\text{pitch\_G})$$

$$(8) \cos(\text{roll\_GP}) * \cos(\text{pitch\_GP}) = \cos(\text{roll\_G}) * \cos(\text{pitch\_G})$$

With

$$(9) A = \sin(\text{pitch\_GP})$$

$$(10) B = -\sin(\text{roll\_GP}) * \cos(\text{pitch\_GP})$$

$$(11) C = \sin(\text{pitch\_G}) \text{ and}$$

$$(12) D = \sin(\text{roll\_G}) * \cos(\text{pitch\_G})$$

and by eliminating  $\cos(\text{yaw})$  between equations (6) and (7), the above equations (6), (7) and (8) can be reduced to

$$(13) \sin(\text{yaw}) = (B * D - A * C) / (B * B + C * C)$$

and by substitution,

$$(14) \cos(\text{yaw}) = (A * B + D * C) / (B * B + C * C)$$

Since the yaw angle is usually small (a couple of degrees at most on ASTRA spacecrafts), equation (13) is often enough to determine the yaw angle and its sign. Therefore, most of the time, only one equation is sufficient. However, equation (8) and (14) can provide additional information to refine the yaw angle.

The yaw angle as measured this way has already been shown to fit the integration of yaw gyro rates, for instance. It will be an invaluable source of attitude information, especially in cases where the knowledge of the yaw angle becomes critical.

The above described method is based on measurement values from two sensors as mostly used in geostationary satellites, for example the ASTRA satellites. However, the principle of the invention can also be used even if measurement values of only two single sensors, for example two single roll sensors or two single pitch sensors are available since two independent equations exist. Indeed, only two of the above given equations (6), (7) and (8) are independent. The

combination of equations (6), (7) and (8) reflects only that the length of the unit vector (0 0 1) is conserved throughout the successive rotations. Hence, two independent equations exist so that two variables could be unknown. It should be noted that the third equation could still be used to help solving the problem. Previously, it had been demonstrated that the problem was solved easily if all variables, except « yaw » were known. In the following, it will be assumed that all variables, except « yaw » and « roll\_GP » are known. In other words, only the pitch angle is measured. However, the location of G is still known in roll and pitch. We can use eq. (6) to determine « yaw ».

Developing system (3), it yields:

$$(15) \quad -\cos(\text{yaw}) * \sin(\text{pitch\_G}) = \\ \sin(\text{yaw}) * \sin(\text{roll\_G}) * \cos(\text{pitch\_G}) - \sin(\text{pitch\_GP})$$

and squaring both sides of equation (15) leads to :

$$(16) \quad [1 - \sin^2(\text{yaw})] * \sin^2(\text{pitch\_G}) = \\ \sin^2(\text{yaw}) * \sin^2(\text{roll\_G}) * \cos^2(\text{pitch\_G}) + \sin^2(\text{pitch\_GP}) \\ - 2 * \sin(\text{yaw}) * \sin(\text{roll\_G}) * \cos(\text{pitch\_G}) * \sin(\text{pitch\_GP})$$

and finally to a quadratic equation in  $\sin(\text{yaw})$  :

$$(17) \quad \sin^2(\text{yaw}) * [\sin^2(\text{pitch\_G}) - \sin^2(\text{roll\_G}) * \cos^2(\text{pitch\_G})] \\ + \sin(\text{yaw}) * [2 * \sin(\text{roll\_G}) * \cos(\text{pitch\_G}) * \sin(\text{pitch\_GP})] \\ + \sin^2(\text{pitch\_G}) - \sin^2(\text{pitch\_GP}) = 0$$

which can easily be solved to determine the yaw angle.

Then, equations (6) and (7) can be used to determine the delta roll angle:

$$(18) \quad \sin(\text{roll\_GP}) = \\ -[\sin(\text{yaw}) * \sin(\text{pitch\_G}) - \\ \cos(\text{yaw}) * \sin(\text{roll\_G}) * \cos(\text{pitch\_G})] / \cos(\text{pitch\_GP})$$

$$(19) \cos(\text{roll\_GP}) = \cos(\text{roll\_G}) * \cos(\text{pitch\_G}) / \cos(\text{pitch\_GP})$$

Obviously, the same kind of demonstration can be done if « roll\_GP » is known and « pitch\_GP » is not.

Thus it has been demonstrated that the yaw angle can be determined with two offset sensors, each one reading only one attitude angle, as long as the reference point of each sensor is known. It should be noted that this method determines « yaw » and « roll\_GP » or « pitch\_GP », the latter ones being deduced from delta angles between the two sensor readings and not representing the real roll or pitch angle. Therefore, if the roll and pitch angles are needed for control, this method cannot determine them in addition to the yaw angle, but at least one of the sensors has to measure both roll and pitch. However, as demonstrated here above, in this particular case, both roll and pitch angles do not have to be measured to determine the yaw angle.

The following general approach will show that actually three measures are necessary to infer the roll, pitch and yaw angles. In other words, one of the sensor has to measure both roll and pitch and one of the sensor could measure either roll or pitch to allow the full determination of the attitude, i.e. roll, pitch and yaw.

Assume the same equations, but this time without any assumption on the value of the angles or on the reference point of the sensors (except being distinct):

For the first sensor:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RM1) & -\sin(RM1) \\ 0 & \sin(RM1) & \cos(RM1) \end{bmatrix} \begin{bmatrix} \cos(PM1) & 0 & \sin(PM1) \\ 0 & 1 & 0 \\ -\sin(PM1) & 0 & \cos(PM1) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

=

$$\begin{bmatrix} \cos(Y) & -\sin(Y) & 0 \\ \sin(Y) & \cos(Y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(R) & -\sin(R) \\ 0 & \sin(R) & \cos(R) \end{bmatrix} \begin{bmatrix} \cos(P) & 0 & \sin(P) \\ 0 & 1 & 0 \\ -\sin(P) & 0 & \cos(P) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RR1) & -\sin(RR1) \\ 0 & \sin(RR1) & \cos(RR1) \end{bmatrix} \begin{bmatrix} \cos(PR1) & 0 & \sin(PR1) \\ 0 & 1 & 0 \\ -\sin(PR1) & 0 & \cos(PR1) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

For the second sensor:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RM2) & -\sin(RM2) \\ 0 & \sin(RM2) & \cos(RM2) \end{bmatrix} \begin{bmatrix} \cos(PM2) & 0 & \sin(PM2) \\ 0 & 1 & 0 \\ -\sin(PM2) & 0 & \cos(PM2) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

=

$$\begin{bmatrix} \cos(Y) & -\sin(Y) & 0 \\ \sin(Y) & \cos(Y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(R) & -\sin(R) \\ 0 & \sin(R) & \cos(R) \end{bmatrix} \begin{bmatrix} \cos(P) & 0 & \sin(P) \\ 0 & 1 & 0 \\ -\sin(P) & 0 & \cos(P) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RR2) & -\sin(RR2) \\ 0 & \sin(RR2) & \cos(RR2) \end{bmatrix} \begin{bmatrix} \cos(PR2) & 0 & \sin(PR2) \\ 0 & 1 & 0 \\ -\sin(PR2) & 0 & \cos(PR2) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

P, R, Y = pitch, roll, yaw angle errors.

RRx, PRx = roll and pitch angles of the sensor's reference point (Depending on the orbit and on the direction to which the sensor points in the spacecraft coordinates; azimuth and elevation of the reference point of sensor 'x' as seen from the spacecraft).

RMx, PMx = measured roll and pitch angles (from center of a star or from the center of earth).

All previous cases presented earlier were actually approximations or particular cases of this last case. Multiplying all these matrices together for both sensors, will provide three equations for each sensors. Again, only two of these equations are independent for each sensor. It provides four independent equations and therefore allows 4 unknowns. This time, the small angle approximation ( $\leq 10^\circ$ ) on the pitch and roll angle errors « P » and « R » is not applied and hence one cannot deduct the readings of both sensors to get around the determination of « R » and « P ».



Therefore, by default, three unknown variables exist: « R », « P » and « Y », what allows one more unknown. This means that for instance one of the following measure could be unknown: RM1, RM2, PM1 or PM2. Hence, in general, in order to fully determine the attitude, one sensor has to measure both roll and pitch and one sensor can measure only the roll or the pitch angle. This is actually not surprising since one has a static geometrical problem with three independent unknown variables and hence one has to measure three independent values.

Obviously the general system given here above can be developed by multiplying all matrices, but the resolution of this system is actually more difficult than the one that was presented earlier. The easiest way would be to solve the corresponding system numerically. A numerical resolution could be developed in another report if needed.



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### Claims

1. Method for determining the yaw angle of a satellite for which a cartesian coordinate system is definable having three axes a first of which is directed substantially tangential to the orbit of the satellite and defines a roll angle (R), a second of which is substantially perpendicular to the orbit plane of the satellite and defines a pitch angle (P), and a third of which is substantially radial to the orbit of the satellite and defines a yaw angle (Y), the method comprising the steps of:

- measuring a first roll angle (R1) and/or a first pitch angle (P1) by means of a first sensor (S1) related to a first reference point (G);
- measuring a second roll angle (R2) and/or a second pitch angle (P2) by means of a second sensor (S2) related to a second reference point (N) which is different from the first reference point (G);
- evaluating the first and/or second roll angles (R1, R2) and the first and/or second pitch angles (P1, P2) to determine the yaw angle (Y) of the satellite.

2. Method according to claim 1, wherein for the first sensor calculations are based on equations representable by the following expression:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RM1) & -\sin(RM1) \\ 0 & \sin(RM1) & \cos(RM1) \end{bmatrix} \begin{bmatrix} \cos(PM1) & 0 & \sin(PM1) \\ 0 & 1 & 0 \\ -\sin(PM1) & 0 & \cos(PM1) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(Y) & -\sin(Y) & 0 \\ \sin(Y) & \cos(Y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(R) & -\sin(R) \\ 0 & \sin(R) & \cos(R) \end{bmatrix} \begin{bmatrix} \cos(P) & 0 & \sin(P) \\ 0 & 1 & 0 \\ -\sin(P) & 0 & \cos(P) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RR1) & -\sin(RR1) \\ 0 & \sin(RR1) & \cos(RR1) \end{bmatrix} \begin{bmatrix} \cos(PR1) & 0 & \sin(PR1) \\ 0 & 1 & 0 \\ -\sin(PR1) & 0 & \cos(PR1) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and wherein for the second sensor calculations are based on equations representable by the following expression:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RM2) & -\sin(RM2) \\ 0 & \sin(RM2) & \cos(RM2) \end{bmatrix} \begin{bmatrix} \cos(PM2) & 0 & \sin(PM2) \\ 0 & 1 & 0 \\ -\sin(PM2) & 0 & \cos(PM2) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(Y) & -\sin(Y) & 0 \\ \sin(Y) & \cos(Y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(R) & -\sin(R) \\ 0 & \sin(R) & \cos(R) \end{bmatrix} \begin{bmatrix} \cos(P) & 0 & \sin(P) \\ 0 & 1 & 0 \\ -\sin(P) & 0 & \cos(P) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(RR2) & -\sin(RR2) \\ 0 & \sin(RR2) & \cos(RR2) \end{bmatrix} \begin{bmatrix} \cos(PR2) & 0 & \sin(PR2) \\ 0 & 1 & 0 \\ -\sin(PR2) & 0 & \cos(PR2) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

with

P, R, Y                      pitch, roll, yaw angle errors,  
 RRx, PRx                    roll and pitch angles of the sensor's  
                                  reference point, and  
 RMx, PMx                    measured roll and pitch angles (from  
                                  center of a star or from the center of  
                                  earth).

3. Method according to claim 1, wherein for both sensors calculations are based on equations representable by the following expression:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\text{roll\_GP}) & -\sin(\text{roll\_GP}) \\ 0 & \sin(\text{roll\_GP}) & \cos(\text{roll\_GP}) \end{bmatrix} \cdot \begin{bmatrix} \cos(\text{pitch\_GP}) & 0 & \sin(\text{pitch\_GP}) \\ 0 & 1 & 0 \\ -\sin(\text{pitch\_GP}) & 0 & \cos(\text{pitch\_GP}) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\text{yaw}) & -\sin(\text{yaw}) & 0 \\ \sin(\text{yaw}) & \cos(\text{yaw}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\text{roll\_G}) & -\sin(\text{roll\_G}) \\ 0 & \sin(\text{roll\_G}) & \cos(\text{roll\_G}) \end{bmatrix} \cdot \begin{bmatrix} \cos(\text{pitch\_G}) & 0 & \sin(\text{pitch\_G}) \\ 0 & 1 & 0 \\ -\sin(\text{pitch\_G}) & 0 & \cos(\text{pitch\_G}) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

with

Roll\_G,

Pitch\_G azimuth and elevation of the reference point (point where the sensor reads ~~reads~~ zero) of one sensor as seen from the spacecraft with the reference point of the other sensor at zero azimuth and zero elevation

Yaw yaw error

Pitch\_GP roll\_G + difference between roll reading on the first sensor and roll reading on the second sensor

Pitch\_GP pitch\_G + difference between pitch reading on the first sensor and pitch reading on the second sensor.

4. Method according to anyone of claims 1, 2 or 3, wherein the first and the second reference points (G, N) are on the earth.
5. Method according to anyone of claims 1 to 4, wherein the first and/or second reference points (G, N) are on a star.
6. Method according to anyone of claims 1 to 5, wherein the satellite is a geostationary satellite.
7. Method for determining an unknown orientation angle of a satellite for which a coordinate system is definable

having three axes which are arranged in an orthogonal trihedron, the method comprising the steps of:

- measuring a first orientation angle (R1) defined by a first axis and/or a second orientation angle (P1) defined by a second axis by means of a first sensor (S1) related to a first reference point (G);
- measuring a third orientation angle (R2) defined by said first axis and/or a fourth orientation angle (P2) defined by said second axis by means of a second sensor (S2) related to a second reference point (N) which is different from the first reference point (G);
- evaluating the first and/or second orientation angles (R1, R2) and the third and/or fourth orientation angles (P1, P2) to determine the unknown orientation angle (Y) of the satellite.

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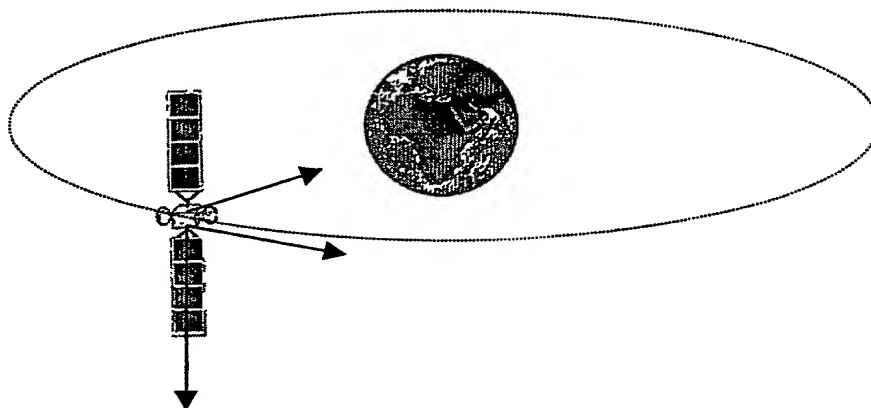


Figure 1

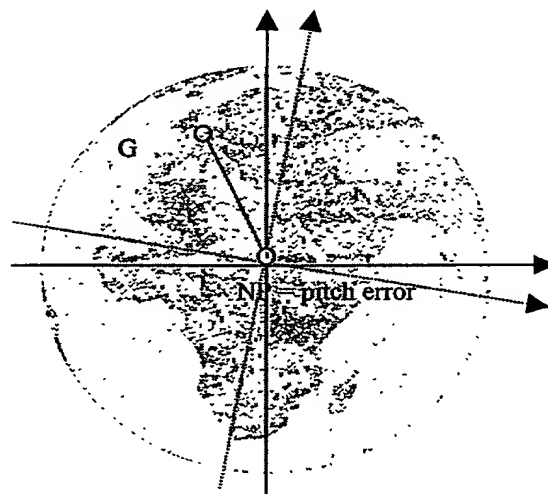


Figure 2

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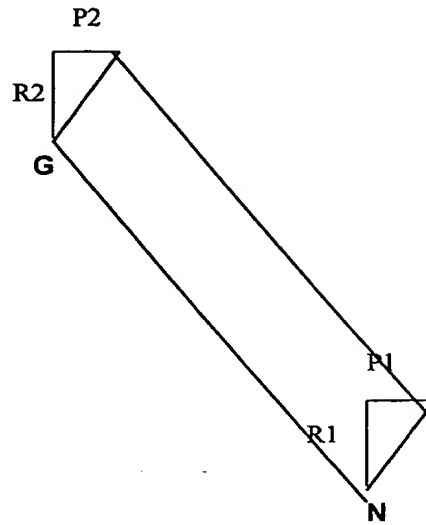


Figure 3

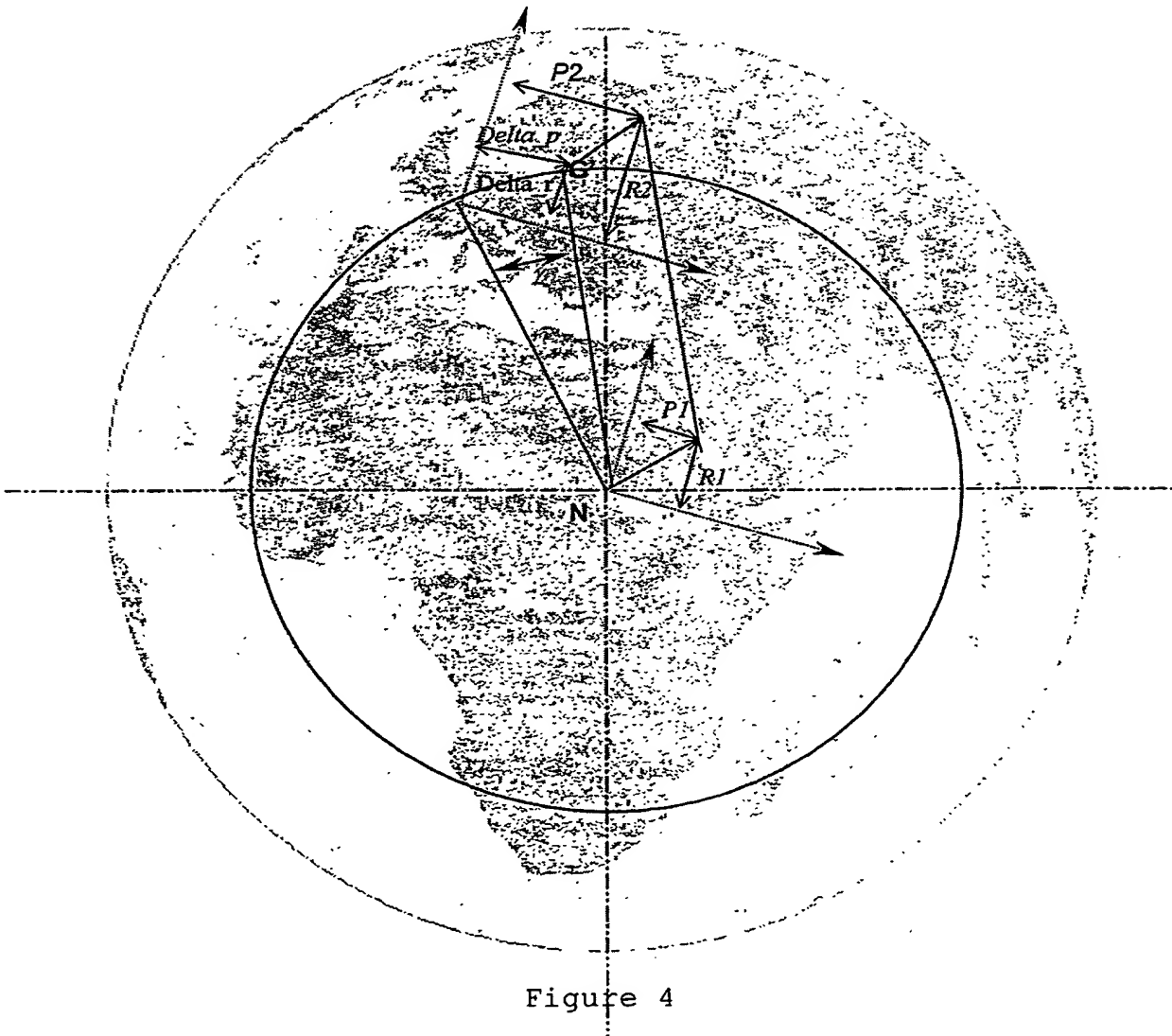


Figure 4



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### Abstract

The method according to the invention allows to determine the yaw angle of a satellite from the reading of two different sensors measuring the roll and/or pitch angles, provided that the reference point of the two sensors are not identical. A description is given basically for geostationary satellites but the method can be applied directly to satellites which are stationary with respect to any star. The method can be employed for circular and non-circular orbits.

Fig. 1

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